

# Diffusion in unstable phases

or

**Can we calculate something that does not  
(and cannot) exist in nature?**

Y. Mishin

Fluids and Materials Program

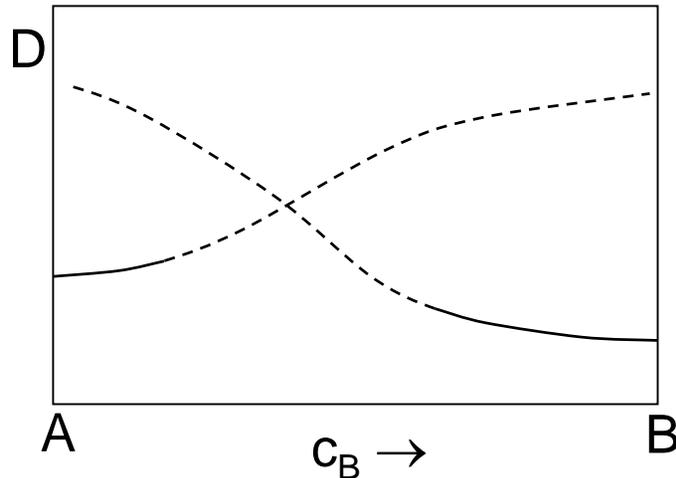
School of Computational Sciences

George Mason University

Fairfax, Virginia

[ymishin@gmu.edu](mailto:ymishin@gmu.edu)

# Problem formulation



How to find  $D_A$  at  $c_B \rightarrow 1$  and  $D_B$  at  $c_B \rightarrow 0$ ?

**Main difficulty:** Element A with the structure of B can be mechanically unstable, and vice versa.

Shear modulus (in GPa) of selected metals

	FCC	BCC
Cu	23.7	-6.8
Ni	43.3	-5.3
Al	26.1	-24.0

Computed with EAM potentials

$$C' = \frac{c_{11} - c_{12}}{2}$$

$C' < 0 \rightarrow$  structure unstable against homogeneous shear deformation

# Problem formulation (cont'd)

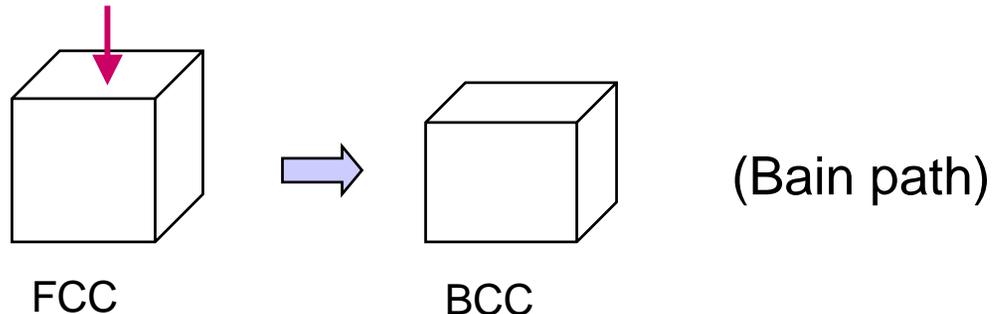
Diffusion calculation in stable phases:

- Vacancy formation energy  $E_v$  by molecular statics
- Vacancy formation entropy  $S_v$  in the harmonic approximation
- Saddle point search by the nudged elastic band (NEB) method -  $E_m$
- Vacancy jump rate in the harmonic TST (Vineyard)

What does not (or may not) work in unstable phases:

- Static relaxation may trigger a transformation to the stable structure
- Some of the normal frequencies are imaginary  $\rightarrow$  harmonic calculations do not work

For example:



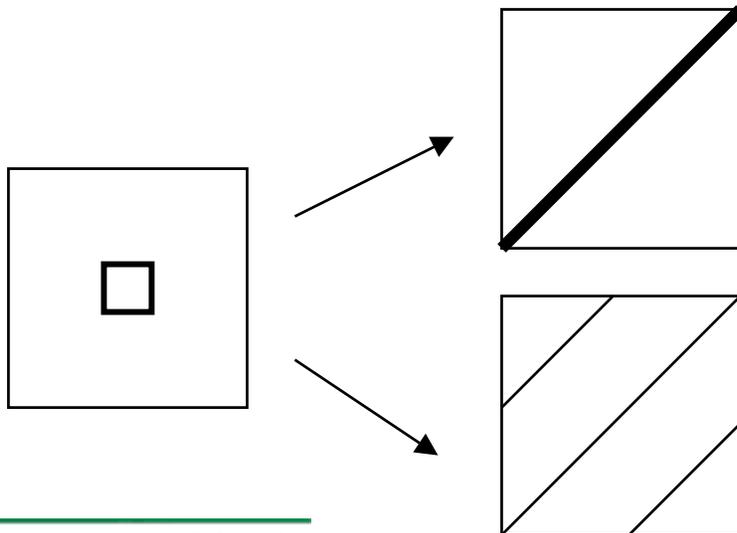
# Approach 1: Constraint by periodic boundaries

**Idea:** Use cubic simulation cell with periodic boundary conditions. The boundary conditions will prevent homogeneous shear deformation.

Tried for Cu, Ni and Al.

**Result:** Static relaxation of a vacancy leads to an infinite “crowdion” formation or trigger stacking-fault generation

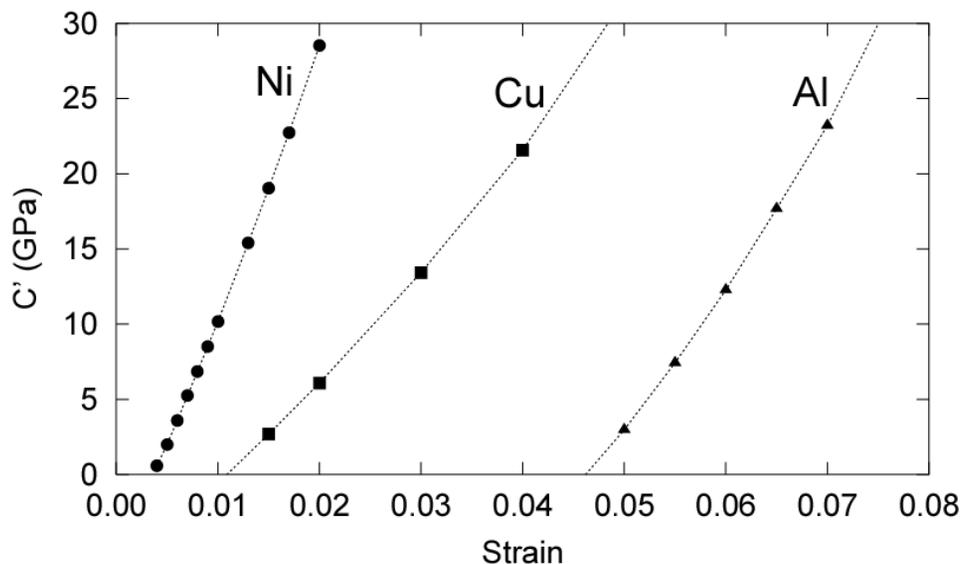
**Conclusion:** It does not work.



## Approach 2: Extrapolation of high-pressure calculations

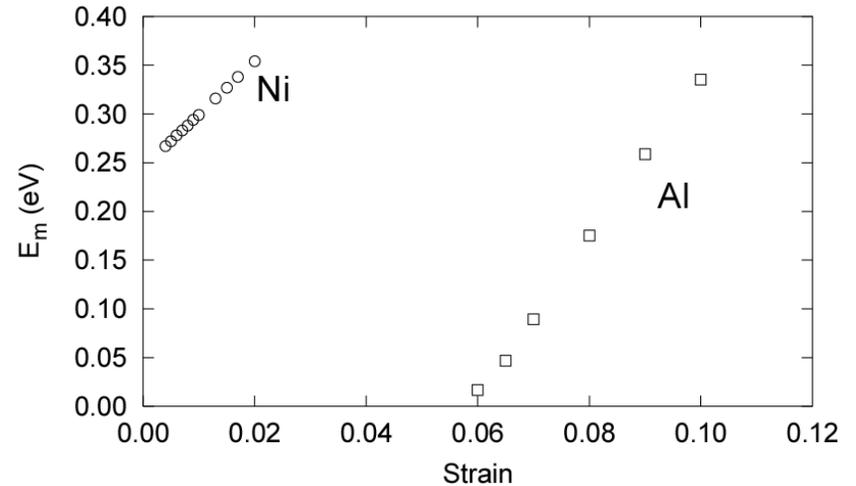
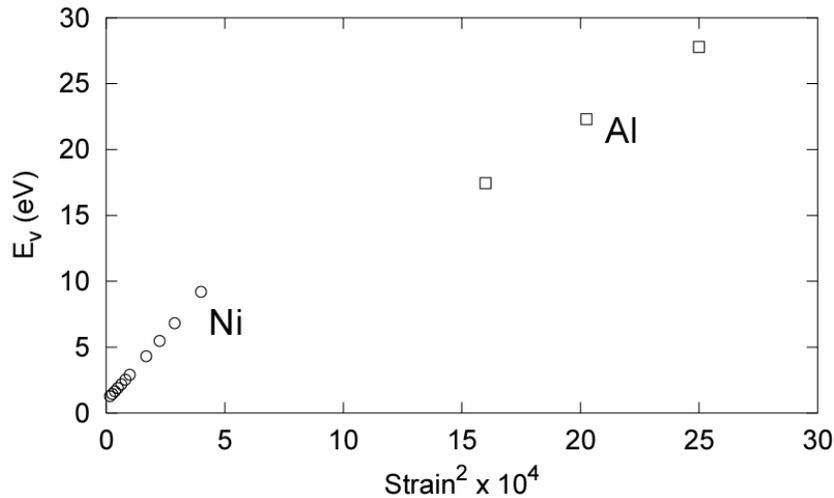
**Idea:** Reverse the sign of  $C'$  by applying a hydrostatic strain  $\varepsilon$ . Calculate  $D$  as a function of  $\varepsilon$ . Extrapolate to  $\varepsilon \rightarrow 0$ .

Tried for Cu, Ni and Al.



Strain stabilizes BCC

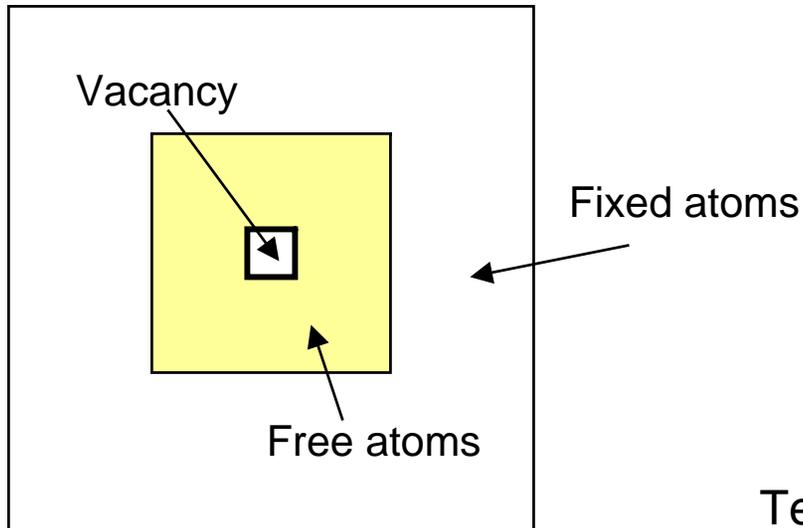
# Vacancy formation and migration



- Vacancy falls apart during relaxation **before**  $C'$  turns to zero. Low-barrier effect?
- $E_m$  can turn to zero when or before  $C'$  turns to zero.
- Calculations are unstable

**Conclusion:** It does not work.

## Approach 3: Constrained relaxation



- Number of free atoms must be small (54 in BCC; 32 in FCC)
- Fixed atoms should stabilize BCC
- Both  $E_v$  and  $E_m$  will be overestimated

Test: calculation for FCC structures

	Cu			Ni			Al		
	Exper	Exact	Approx	Exper	Exact	Approx	Exper	Exact	Approx
$E_v$ (eV)	1.28	1.272	1.282	1.60	1.57	1.58	0.68	0.71	0.72
$E_m$ (eV)	0.71	0.70	0.74	1.30	1.19	1.27	0.65	0.65	0.69

➡ Works well for FCC

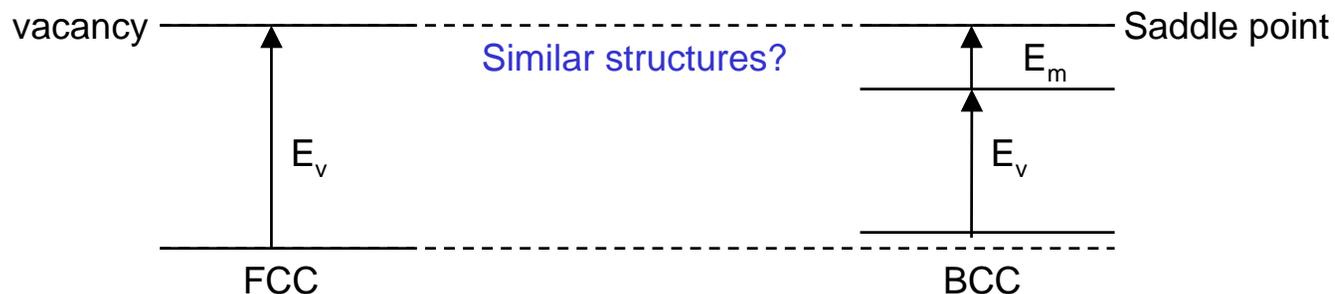
## Does it work for BCC?

	Cu	Ni	Al
$E_v$ (eV)	0.96	1.12	0.30
$E_m$ (eV)	0.32	0.47	0.16
$Q_{\text{BCC}}$ (eV)	1.28	1.59	0.46
$Q_{\text{FCC}}$ (eV)	2.02	2.85	1.41
$Q_{\text{BCC}} / Q_{\text{FCC}}$	0.74	0.56	0.33

**Conclusion:** It has a potential. Further testing is needed (other metals, more accurate potentials,...).

### As an aside:

For Cu and Ni,  $Q_{\text{BCC}} \approx (E_v)_{\text{FCC}}$ . Coincidence or...



# Diffusion in ordered compounds

- We need to know diffusion mechanisms in ordered phases:
  - Predictive calculations
  - CALPHAD-type diffusion calculations
  - Interpretation of experiment
- History:
  - Diffusion mechanisms are complex
  - Experimental studies are difficult
  - Several mechanisms were proposed in 1970-80s (6-jump cycles, divacancy, etc.) but verification by simulations was impossible
  - Understanding is still poor
- New capabilities of atomistic simulations (accurate potentials, advanced saddle-point search, etc.). There is some progress in understanding: direct observation of 6-jump cycles, discovery of collective diffusive jumps, etc.
- New compounds become technologically important (e.g. silicides, etc.).
- **There has never been a better time to revisit the problem by theory and experiment.**

# Creep in ordered compounds

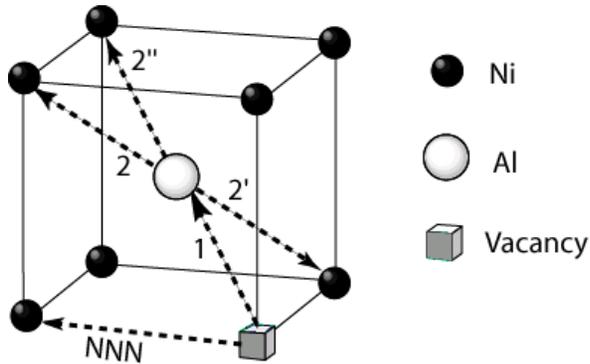
- Key question: What kind of diffusion coefficient ( $D_c$ ) controls diffusion creep? [Generally,  $D_c \neq D_{\text{inter}}$ ]

Example: for a strongly ordered compound: 
$$\frac{1}{D_c} = \frac{c_A}{D_A} + \frac{c_B}{D_B}$$

(similar to ambipolar diffusion in ionics)

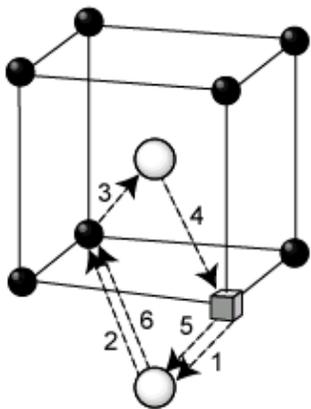
- Creep in disordered alloys (de-alloying, slow species control, etc.)
- What happens in partially disordered compounds?

# Diffusion in B2-NiAl

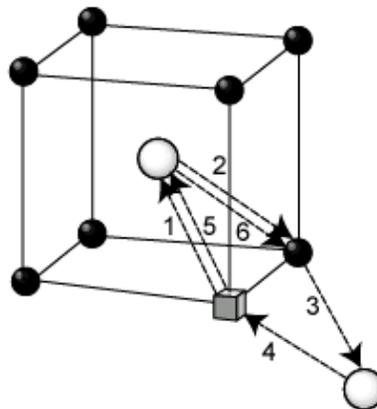


- Ni and Al isolated from each other
- NNN jumps on Ni sublattice
- NN vacancy jumps create mechanically unstable configurations
- Collective two-atom jumps
- Cyclic mechanisms

[110] cycle



[100] straight cycle



[100] bent cycle

